EXAM – BATXILERGOA 1: Energy - Electricity	
Name:	**Group:**

1	**We let a body move down a slope without initial speed. The height at the beginning is 30 m and the angle of the slope is 30°. At the lowest point the speed of the body is 20 m/s. Find the friction coefficient between the body and the floor.**

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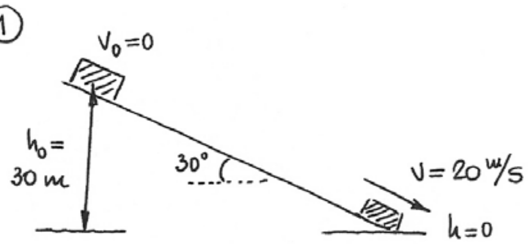


μ?

The strategy to solve the problem can be:

$\Delta E = W_{F,F_f}$

$\overset{a}{\longrightarrow} F_f \overset{N}{\underset{b}{\longleftrightarrow}} \mu$

a) Determine the work done by non-conservative forces and the friction force

b) Find the coefficient of friction

a Determination of the friction force

$E_{initial} = KE_i + PE_i = \frac{1}{2} m v_0^2 + m g h_0 \xrightarrow{V_0=0} E_{initial} = m \times 10 \frac{m}{s^2} \times 30\,m \rightarrow$

$\rightarrow E_{initial} = m \times 300 \frac{m^2}{s^2}$

$E_{final} = KE_f + PE_f = \frac{1}{2} m v^2 + m g h \xrightarrow{h=0} E_{final} = \frac{1}{2} \times m \times \left(20 \frac{m}{s}\right)^2 \rightarrow$

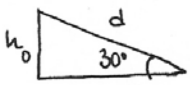
$\rightarrow E_{final} = m \times 200 \frac{m^2}{s^2}$

$\Delta E = E_{final} - E_{initial} = \left(m \times 200 \frac{m^2}{s^2}\right) - \left(m \times 300 \frac{m^2}{s^2}\right) = - m \times 100 \frac{m^2}{s^2}$

According to the conservation of mechanical energy:

$\Delta E = W_{F,F_f} \rightarrow - m \times 100 \frac{m^2}{s^2} = F_f \times d \times \cos 180^0$

The distance travelled by the body is:



$\sin 30^0 = 0.5 = \frac{h_0}{d} \rightarrow d = \frac{h_0}{0.5} = \frac{30\,m}{0.5} \rightarrow d = 60\,m$

The friction force can take this form (we can't find the value at this stage):

$- m \times 100 \frac{m^2}{s^2} = F_f \times 60\,m \times (-1) \rightarrow F_f = m \times \frac{100}{60} \frac{m}{s^2}$

On the other hand, we can express the friction force as:

$$F_f = \mu \times N \longrightarrow N = W_y = m \times g \times \cos 30^\circ = m \times 10 \frac{m}{s^2} \times 0.87 =$$
$$= 8.7 \frac{m}{s^2} \times m$$

Therefore, we have two expressions for the friction force:

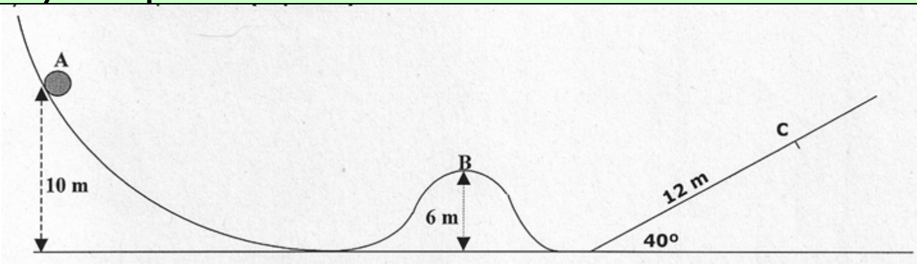
$$\begin{cases} F_f = m \times \frac{100}{60} \frac{m}{s^2} & \text{(derived from the conservation of mechanical energy)} \\ F_f = \mu \times m \times 8.7 \frac{m}{s^2} & \text{(derived from the definition of the friction force)} \end{cases}$$

(b)

Equalizing both expressions we have:

$$m \times \frac{100}{60} \frac{m}{s^2} = \mu \times m \times 8.7 \frac{m}{s^2} \rightarrow \mu = \frac{100}{60 \times 8.7} \rightarrow \boxed{\mu = 0.19}$$

2	Suppose that the friction is negligible in this exercise. If we let the body move, without initial speed, determine: a) the speed at "B" b) the speed at "C"



② ⓐ From $A \to B$

There are no $\begin{cases}\text{friction force} \\ \text{external force}\end{cases} \to W_{F, F_f} = 0$

Therefore, $E_A = E_B$

$E_A = KE_A + PE_A = \frac{1}{2} m v_A^2 + mgh_A \xrightarrow[h_A = 10\,m]{v_A = 0}$

$\to E_A = m \times 10 \frac{m}{s^2} \times 10\,m = m \times 100 \frac{m^2}{s^2}$

$E_B = KE_B + PE_B = \frac{1}{2} m v_B^2 + mgh_B \xrightarrow{h_B = 6\,m}$

$\to E_B = \frac{1}{2} m v_B^2 + m \times 10 \frac{m}{s^2} \times 6\,m = \frac{1}{2} m v_B^2 + m \times 60 \frac{m^2}{s^2}$

From here, we will find the speed at B:

$E_A = E_B \to m \times 100 \frac{m^2}{s^2} = \frac{1}{2} m v_B^2 + m \times 60 \frac{m^2}{s^2} \to$

$\to 100 \frac{m^2}{s^2} = \frac{1}{2} v_B^2 + 60 \frac{m^2}{s^2} \to v_B^2 = 80 \frac{m^2}{s^2} \to \boxed{v_B = 8.94 \frac{m}{s}}$

ⓑ From $A \to C$

$E_A = E_C$

$E_C = KE_C + PE_C = \frac{1}{2} m v_C^2 + mgh_C$

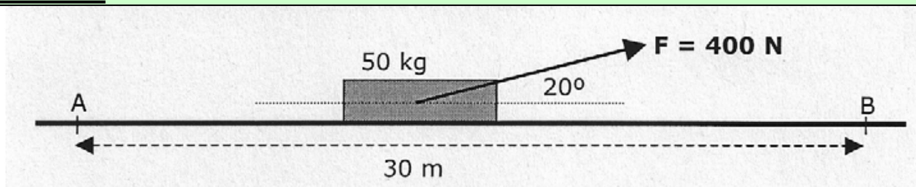
$h_C = 12\,m \times \sin 40^\circ = 7.71\,m$

$E_C = \frac{1}{2} m v_C^2 + m \times 10 \frac{m}{s^2} \times 7.71\,m = \frac{1}{2} m v_C^2 + m \times 77.1 \frac{m^2}{s^2}$

The speed at C is:

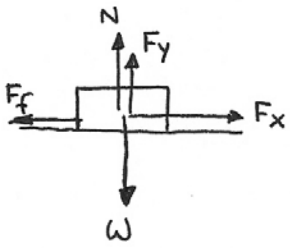
$E_A = E_C \to m \times 100 \frac{m^2}{s^2} = \frac{1}{2} m v_C^2 + m \times 77.1 \frac{m^2}{s^2} \to$

$\to \frac{1}{2} v_C^2 = 22.9 \frac{m^2}{s^2} \to v_C = \sqrt{2 \times 22.9 \frac{m^2}{s^2}} \to \boxed{v_C = 6.77 \frac{m}{s}}$

3	**The friction coefficient between the body and the floor is 0.25 in this case. The body was still at point A and started to move because a F force was applied from A to B. Find the speed at B, using the <u>WORK-ENERGY</u> theorem**
	

③ The equation of the Work-energy theorem is:

$$W_{TOTAL} = \Delta KE = KE_f - KE_i$$



The values of the forces are:

$$W = m \times g = 500\,N$$

$$F_x = F \times \cos 20^\circ = 400\,N \times \cos 20^\circ = 376\,N$$

$$F_y = F \times \sin 20^\circ = 400\,N \times \sin 20^\circ = 137\,N$$

$$N + F_y = W \rightarrow N = W - F_y = 363\,N$$

$$F_f = \mu \times N = 0.25 \times 363\,N = 91\,N$$

The total work is:

$$W_{TOTAL} = \underset{0}{\underset{\shortparallel}{W_W}} + W_{F_x} + \underset{0}{\underset{\shortparallel}{W_{F_y}}} + \underset{0}{\underset{\shortparallel}{W_N}} + W_{F_f}$$

$$W_{TOTAL} = 376N \times 30m \times \cos 0^\circ + 91N \times 30m \times \cos 180^\circ$$

$$W_{TOTAL} = 11280\,J - 2730\,J = 8550\,J$$

Applying the work-energy theorem:

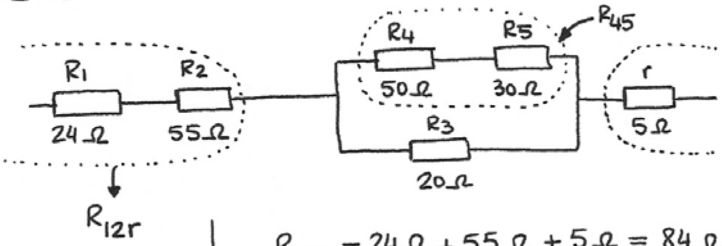
$$W_{TOTAL} = \Delta KE = KE_f - KE_i \xrightarrow[V_0 = 0 \rightarrow KE_i = 0]{} W_{TOTAL} = KE_f \rightarrow$$

$$\rightarrow 8550\,J = \frac{1}{2} \times m \times V_f^2 \rightarrow 8550\,J = \frac{1}{2} \times 50\,Kg \times V_f^2 \rightarrow$$

$$\rightarrow \quad V_f = \sqrt{\frac{2 \times 8550\,J}{50\,Kg}} \rightarrow \boxed{V_f = 18.5\,\frac{m}{s}}$$

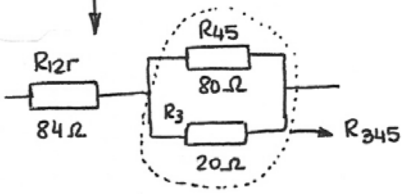
4	**In this circuit, determine** **a) the equivalent resistance** **b) the electric power of the battery** **c) the potential difference between “a” and “b”**

4 a) The equivalent resistance of the circuit (R_{equiv})



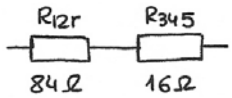
$R_{12r} = 24\,\Omega + 55\,\Omega + 5\,\Omega = 84\,\Omega$

$R_{45} = 50\,\Omega + 30\,\Omega = 80\,\Omega$



$$\frac{1}{R_{345}} = \frac{1}{R_3} + \frac{1}{R_{45}} = \frac{1}{20\,\Omega} + \frac{1}{80\,\Omega} = \frac{4+1}{80\,\Omega}$$

$$\frac{1}{R_{345}} = \frac{5}{80\,\Omega} \rightarrow R_{345} = \frac{80\,\Omega}{5} = 16\,\Omega$$



$\rightarrow R_{equiv} = R_{12r} + R_{345} \rightarrow \boxed{R_{equiv} = 100\,\Omega}$

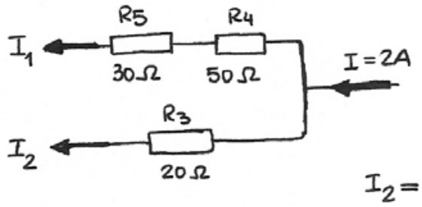
b) Electric power of the battery (P)

$$I = \frac{\varepsilon}{R_{equiv}} \rightarrow I = \frac{200\,V}{100\,\Omega} = 2\,A$$

$P = \varepsilon \times I = 200\,V \times 2A \rightarrow \boxed{P = 400\,W}$

c) The potential difference between "a" and "b"

We need to know the current through the upper branch



The system is:

$$\begin{cases} I_1 + I_2 = 2 \\ I_1 \times 80 = I_2 \times 20 \end{cases} \rightarrow$$

$$I_2 = \frac{80 \times I_1}{20} = 4 \times I_1 \rightarrow I_1 + 4I_1 = 2$$

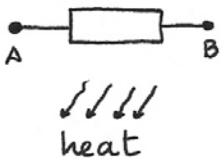
$$I_1 = \frac{2A}{5} = 0.4\,A$$

$V_a - V_b = 2A \times 55\,\Omega + 0.4\,A \times 50\,\Omega = 110\,V + 20\,V$

$\boxed{V_a - V_b = 130\,V}$

5	**The power of an electric heater is 1760 W and is connected to a 220 V supply line. Determine:** **a) the resistance of the heater** **b) the energy dissipated in 6 hrs (in KW-h)**
1 KW-h = 3 600 000 J	

⑤ ⓐ The resistance of the heater



$V_A - V_B = I \times R \rightarrow 220\,V = I \times R$

$P = I^2 \times R \rightarrow 1760\,W = I^2 \times R$

We have this system of two equations:

$$\begin{cases} 220 = I \times R \\ 1760 = I^2 \times R \end{cases} \longrightarrow \frac{1760}{220} = \frac{I^2 \times R}{I \times R} = I$$

$I = 8A$

$$R = \frac{220V}{8A} \rightarrow \boxed{R = 27.5\,\Omega}$$

ⓑ The energy dissipated in 6 hrs

$$E = P \times t \rightarrow E = 1760\,\frac{J}{s} \times 6h \times \frac{3600\,s}{1\,h}$$

$E = 38\,016\,000\,J$

$$E = 38\,016\,000\,J\ \frac{1\,Kw\text{-}h}{3\,600\,000\,J} \rightarrow \boxed{E = 10.56\ Kw\text{-}h}$$