(1) (a) Types of motion:

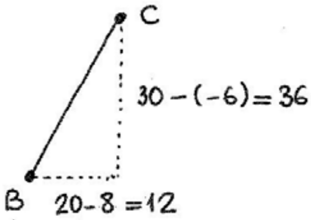
A–B ... motion with constant velocity

B–C ... motion with constant acceleration
acceleration $\rightarrow a>0$ positive

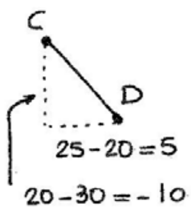
C–D ... motion with constant acceleration
acceleration $\rightarrow a<0$ negative

(b) The values of acceleration
acceleration = slope

$a_1 = \text{slope} = 0$

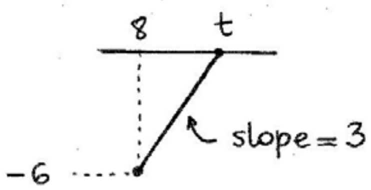


$$a_2 = \text{slope} = \frac{36}{12} = 3\ \frac{m}{s^2}$$



$$a_3 = \text{slope} = \frac{-10}{5} = -2\ \frac{m}{s^2}$$

(c) change in direction $\rightarrow v = 0$



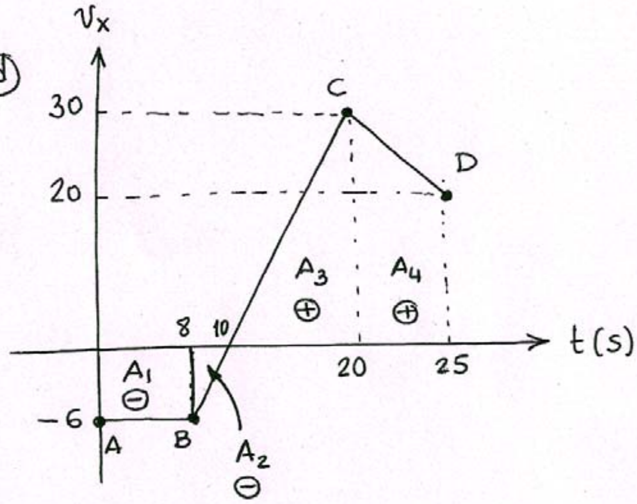
$$\text{slope} = \frac{v - v_0}{t - t_0}$$

$$\downarrow$$

$$3 = \frac{0-(-6)}{t-8} = \frac{6}{t-8} \rightarrow$$

$$\rightarrow t - 8 = \frac{6}{3} = 2 \rightarrow t = 10\ s$$

① ⓓ



displacement = area

$\Delta x_1 = A_1 = (-6) \times 8 = -48\ m$

$\Delta x_2 = A_2 = \frac{1}{2} \times (10-8) \times (-6) = -6\ m$

$\Delta x_3 = A_3 = \frac{1}{2}\ (20-10) \times 30 = 150\ m$

$\Delta x_4 = A_4 = \frac{1}{2}\ (25-20) \times (30-20) + (25-20) \times 20 =$

$= 25\ m + 100\ m = 125\ m$

$\Delta x = -48\ m - 6\ m + 150\ m + 125\ m = 221\ m$

ⓔ $x = x_0 + \Delta x$

$x_0 = -18\ m$

$\Delta x = -48\ m - 6\ m + 150\ m = 96\ m$

$x = -18\ m + 96\ m = 78\ m \longleftrightarrow \vec{r} = 78\ \vec{i}\ (m)$

② (a) The equations of the positions vectors are

$$v_{0A} = 180\,\frac{km}{h}\,\frac{1\,h}{3600\,s}\,\frac{1000\,m}{1\,Km} = 50\,\frac{m}{s}$$

$$v_{0B} = 72\,\frac{km}{h}\,\frac{1\,h}{3600\,s}\,\frac{1000\,m}{1\,Km} = 20\,\frac{m}{s}$$

A: $x_0 = -500\,m$; $v_0 = 50\,\frac{m}{s}$; $a = -2\,\frac{m}{s^2}$

$$\boxed{\vec{r}_A = (-500 + 50t - t^2)\,\vec{i}\ (m)}$$

B: $x_0 = 500\,m$; $v_0 = -20\,\frac{m}{s}$

$$\boxed{\vec{r}_B = (500 - 20t)\,\vec{i}\ (m)}$$

(b) The first time they pass each other

↳ $x_A = x_B$ ... they are at the same position

$$-500 + 50t - t^2 = 500 - 20t$$

$$t^2 - 70t + 1000 = 0$$

$$t = \frac{70 \pm \sqrt{70^2 - 4000}}{2} = \frac{70 \pm 30}{2}$$

↗ $t = 50\,s$

↘ $\boxed{t = 20\,s}$ ... the first time

$$x_A = x_B = 500 - 20 \times 20 = \boxed{100\,m}$$

(c) "A" changes its direction ... $v = 0$

$$x = -500 + 50t - t^2 \xrightarrow{\text{differentiation}} v = 50 - 2t$$

$$v = 50 - 2t = 0 \longrightarrow \boxed{t = 25\,s}$$

(d) The distance between them when $t = 12\,s$

$$x_A = -500 + 50t - t^2 \xrightarrow{t=12\,s} x_A = -44\,m$$

$$x_B = 500 - 20\,t \xrightarrow{t=12\,s} x_B = 260\,m$$

$$d = x_B - x_A = 260\,m - (-44\,m) = 304\,m$$

(e) A crosses the origin ($x=0$) towards the left ($v<0$)

$$-500+50t-t^2=0 \longleftrightarrow t^2-50t+500=0$$

$$t=\frac{50\pm\sqrt{50^2-2000}}{2}=\frac{50\pm 22.36}{2}$$

$t = 13.82$ s

$t = 36.18$ s

- when $t=13.82$ s $\longrightarrow v=50-2t>0$ towards the right
- when $\boxed{t=36.18\text{ s}}$ $\longrightarrow v=50-2t<0$ towards the left

$$v=50-2\times 36.18=\boxed{-22.36\ \frac{m}{s}}$$

③ ⓐ The equation of the position vector

$$\boxed{\vec{r} = (60 + 40t - 5t^2)\,\vec{j} \ \ (m)}$$

ⓑ The equation of the velocity, by differentiation

$$\boxed{\vec{v} = (40 - 10t)\,\vec{j} \ \ (m/s)}$$

ⓒ The maximum height is reached when $v = 0$

$$40 - 10t = 0 \longrightarrow t = \frac{40}{10} = 4\ s$$

$$y = 60 + 40t - 5t^2 \xrightarrow{t=4s} \boxed{y = 140\ m}$$

ⓓ When the body hits the floor $\longrightarrow y = 0$

$$y = 60 + 40t - 5t^2 = 0$$

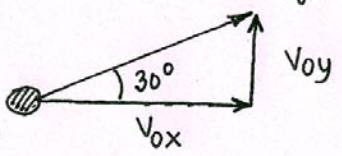
$$t^2 - 8t - 12 = 0$$

$$t = \frac{8 \pm \sqrt{64 + 48}}{2} = \frac{8 \pm 10.58}{2} \begin{cases} t < 0 \\ t = 9.29\ s \end{cases}$$

$$\vec{v} = (40 - 10t)\,\vec{j} \ (m/s)$$

$$\xrightarrow[t=9.29s]{} \boxed{\vec{v} = -52.9\,\vec{j} \ (m/s)}$$

④ ⓐ The components of the initial velocity are



$$V_{0y} = V_0 \cdot \sin\alpha = 150\,\frac{m}{s}\cdot\frac{1}{2} = \boxed{75\, m/s}$$

$$V_{0x} = V_0 \cdot \cos\alpha = 150\,\frac{m}{s}\cdot\frac{\sqrt{3}}{2} = \boxed{129.9\, m/s}$$

ⓑ The equation of the position vector

$$\boxed{\vec{r} = 129.9\,t\,\vec{i} + (40 + 75t - 5t^2)\,\vec{j}\ \ (m)}$$

ⓒ The equation of the velocity

$$\vec{r} \xrightarrow{\text{differentiation}} \vec{v}$$

$$\boxed{\vec{v} = 129.9\,\vec{i} + (75 - 10\,t)\,\vec{j}\ \ (m/s)}$$

ⓓ The maximum height $\longrightarrow V_y = 0$

$$V_y = 75 - 10t = 0 \longrightarrow t = \frac{75}{10} = 7.5\,s$$

$$y = 40 + (75 \times 7.5) - (5 \times 7.5^2) = 40 + 562.5 - 281.25$$

$$\boxed{y_{max} = 321.25\, m}$$

ⓔ it hits the floor $\longrightarrow y = 0$

$$y = 40 + 75t - 5t^2 = 0 \longrightarrow t^2 - 15t - 8 = 0$$

$$t = \frac{15 \pm \sqrt{15^2 + 32}}{2} = \frac{15 \pm 16.03}{2} \quad \begin{cases} t < 0 \\ t = 15.52\, s \end{cases}$$

$$x = 129.9\,t \xrightarrow{t = 15.52\,s} \boxed{x = 2\,016\, m}$$