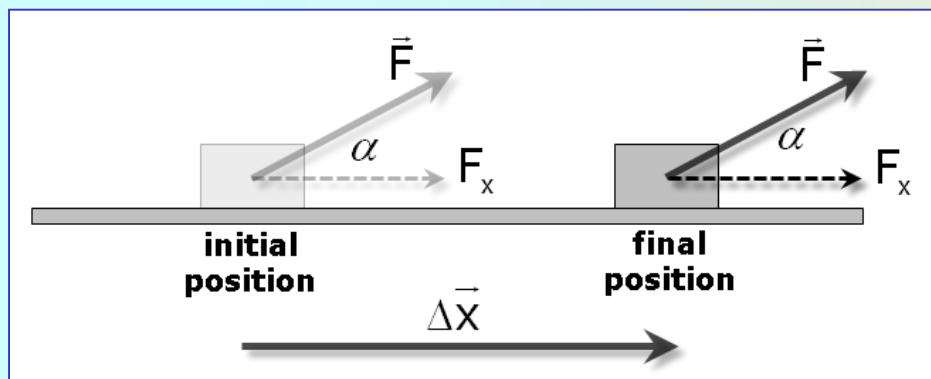


Mechanical Work

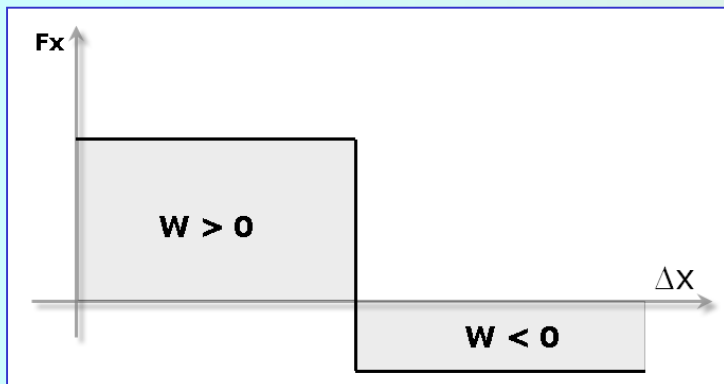


Definition

In the physical sciences the meaning of work is the following: if we exert a force \mathbf{F} on an object causing it to move a distance $\Delta \mathbf{x}$, then the work done by the force is defined to be the scalar product (dot product):

$$W = \vec{F} \cdot \Delta \vec{X} = F \cdot \Delta x \cdot \cos \alpha$$

where \mathbf{F} and $\Delta \mathbf{x}$ are vectors (the vector $\Delta \mathbf{x}$ indicates the direction of motion)



$$W = \vec{F} \cdot \Delta \vec{X} \xrightarrow{F_x = F \cdot \cos \alpha} W = F_x \cdot \Delta X$$

The SI unit for work is the **joule (J)**

$$1 \text{ joule (J)} = 1 \text{ N} \times 1 \text{ m} = 1 \frac{\text{kg} \times \text{m}^2}{\text{s}^2}$$

The work done by a force on a body can be calculated graphically: the work is the area below the $F_x - \Delta x$ graphic.

Mechanical Work

Problem #1

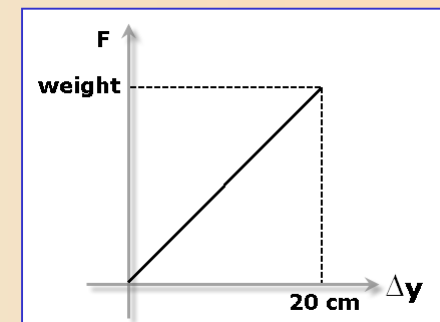
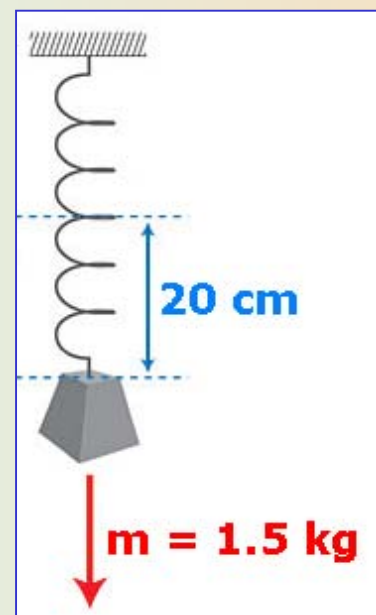
A child pulls a toy 3.0 m across the floor by a string, applying a force of constant magnitude 8 N.

During the first two meters the string is parallel to the floor. During the third meter the string makes an angle of 30° with the horizontal direction.

What is the total work done by the string on the toy? Determine it using the equation and graphically.

Problem #2

Calculate the work done on a spring by the suspended weight in the picture



Mechanical Work

Problem #1

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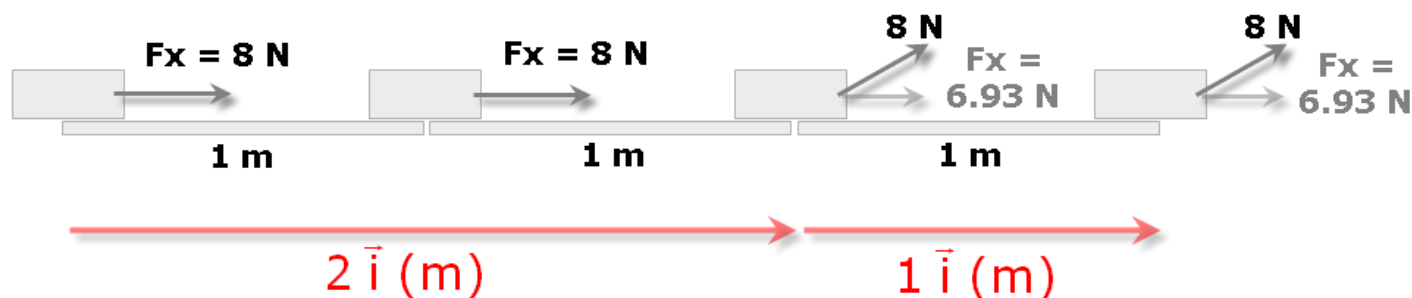
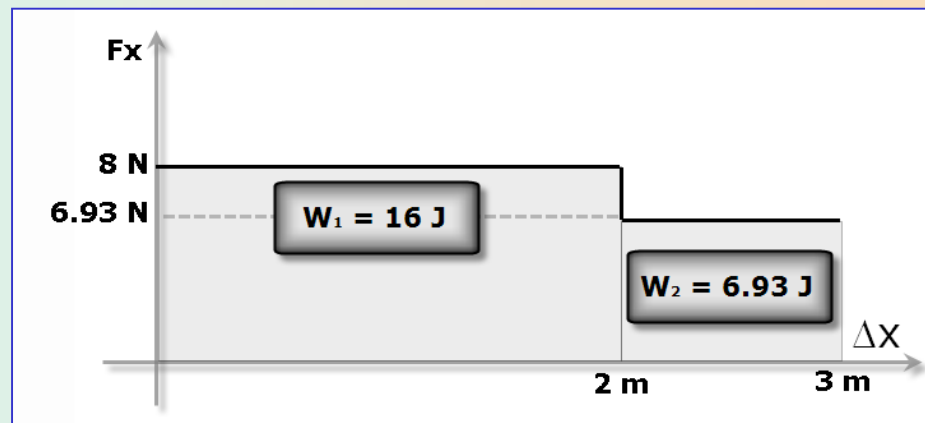
The work is the dot product of force and distance:

$$W = \vec{F} \cdot \Delta\vec{X} = F \cdot \Delta X \cdot \cos \alpha$$

In our case:

$$W = 8 \vec{i} \text{ (N)} \times 2 \vec{i} \text{ (m)} + 6.93 \vec{i} \text{ (N)} \times 1 \vec{i} \text{ (m)} = 22.93 \text{ J}$$

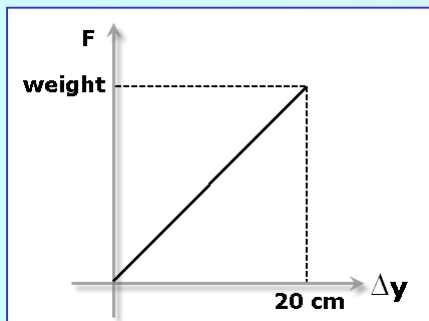
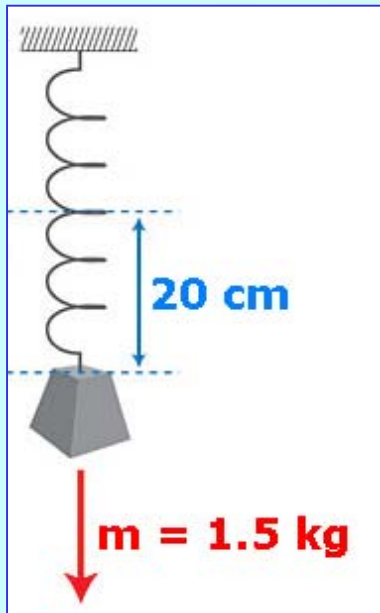
Graphically, we get the same result (by calculating the area):



Mechanical Work

Problem #2

Calculate the work done on a spring by the suspended weight in the picture



The shortest way to calculate the work done on the spring is graphically:

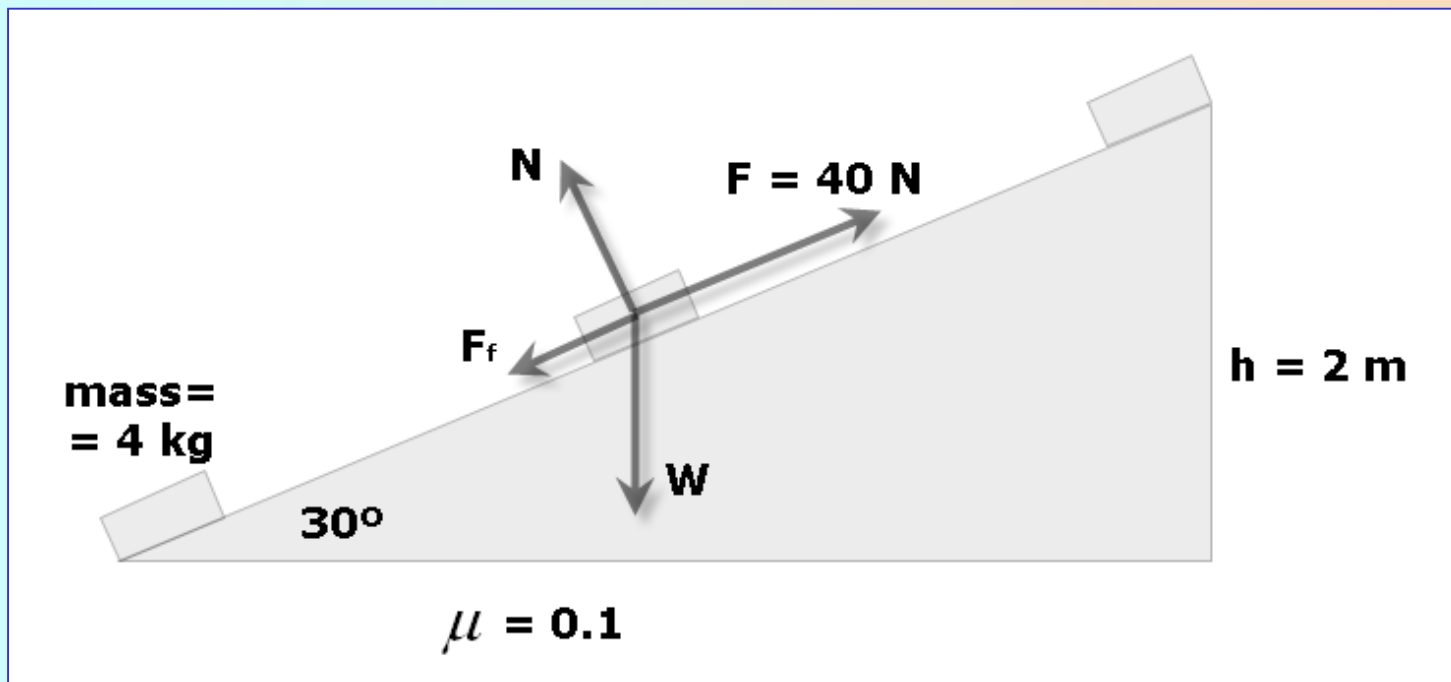
$$W (\text{weight}) = 1.5 \text{ kg} * 10 \text{ m/s}^2 = 15 \text{ N}$$

$$A (\text{area}) = W (\text{work}) = \frac{1}{2} * 0.2 \text{ m} * 15 \text{ N} = 1.5 \text{ J}$$

Mechanical Work

Problem #3

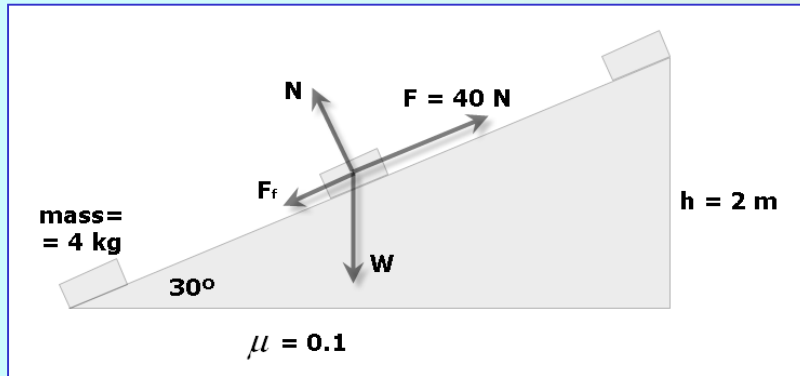
Calculate the work done by each force on the body and the total work.



Mechanical Work

Problem #3

Calculate the work done by each force on the body and the total work.



The distance travelled by the body is:

$$\sin 30^\circ = 0.5 = \frac{h}{\Delta x} = \frac{2 \text{ m}}{\Delta x} \rightarrow$$

$$\Delta x = 4 \text{ m}$$

The forces are:

$$W \text{ (weight)} = 40 \text{ N}$$

$$N = W_y = 40 \text{ N} * \cos 30^\circ = 34.6 \text{ N}$$

$$F_f = \mu * N = 3.46 \text{ N}$$

The work done by each force:

$$W_N = 34.6 \text{ N} * 4 \text{ m} * \cos 90^\circ = 0$$

$$W_W = 40 \text{ N} * 4 \text{ m} * \cos 120^\circ = -80 \text{ J}$$

$$W_{F_f} = 3.46 \text{ N} * 4 \text{ m} * \cos 180^\circ = -13.84 \text{ J}$$

$$W_F = 40 \text{ N} * 4 \text{ m} * \cos 0^\circ = 160 \text{ J}$$

Notice that:

-the work done by the normal force is always 0

-the angle between the direction of motion and the friction force is always 180° ($\cos 180^\circ = -1$)

The total work is:

$$W_{\text{total}} = W_N + W_W + W_{F_f} + W_F =$$

$$-80 \text{ J} - 13.84 \text{ J} + 160 \text{ J} = 66.16 \text{ J}$$