Topic:	Position, distance, displacement
Objective:	FK_11_01

Given the positions of a body in a graphic at different times
the student must be capable of doing the following:

- **determine the expressions of the position vectors**
- **calculate the distance from the origin**
- **determine the displacement between two different moments**

Position vector

The position vector is the vector from the origin of the coordinate system O(0,0) to the position of the body P(x,y). It is shown as the vector $\vec{r}$.

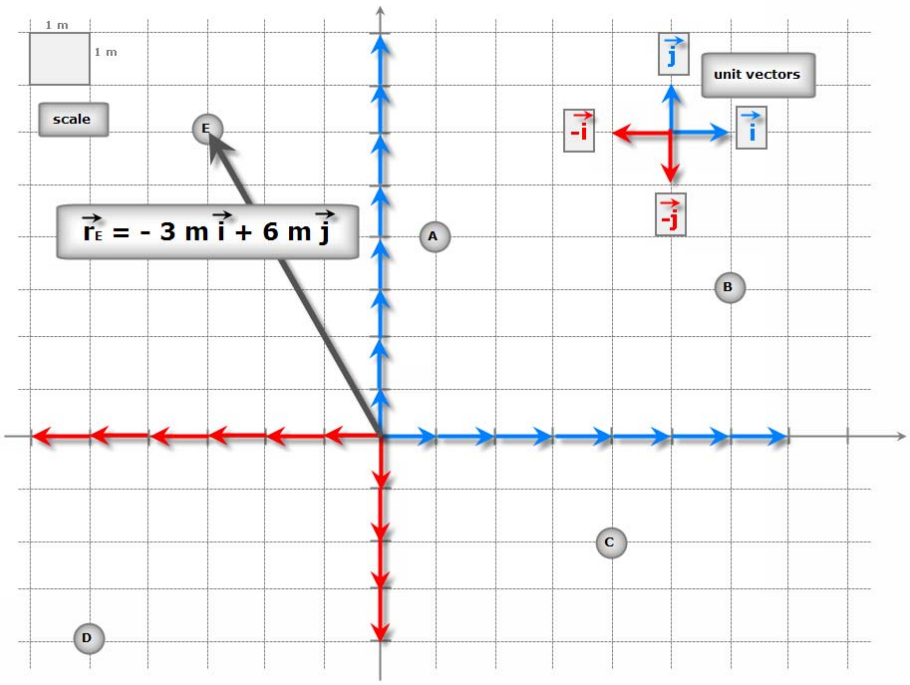
The vectors of a rectangular x-y coordinate system can be expressed by the combination of the unit vectors $\vec{i}$ and $\vec{j}$ along the x and y directions, respectively.

In our example, we can express the position vector of "E" as a combination of both vectors:

$\vec{r}_E$ ... 3 m to the left & 6 m upwards

↓

$\vec{r}_E = -3\ m\ \vec{i} + 6\ m\ \vec{j}$



Exercise

Draw the rest of the position vectors and determine their expressions:

$\vec{r}_A$, $\vec{r}_B$, $\vec{r}_C$, $\vec{r}_D$

Components of the position vector

The position vector (or any other vector) can be seen as a sum of two vectors: one along the “x” axis and other along the “y” axis.

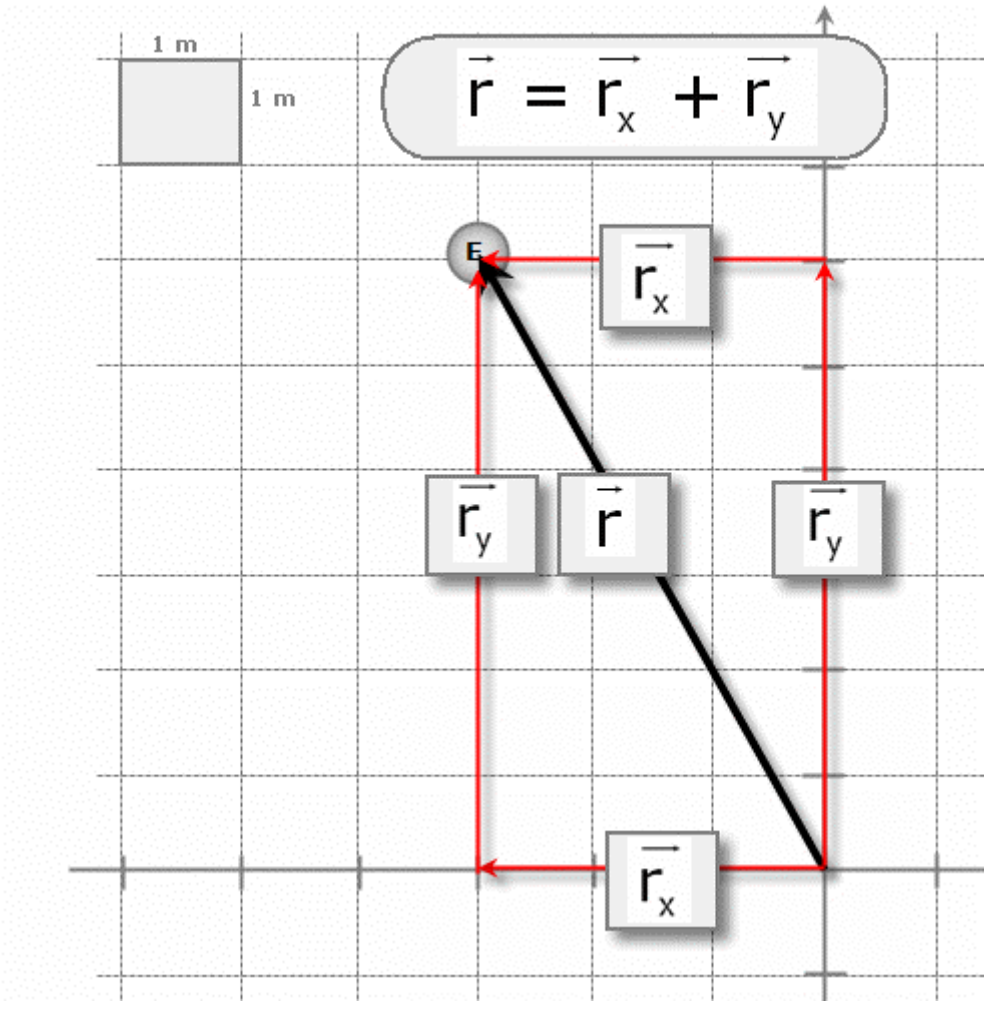
Therefore, we can say that:

$\vec{r} = \vec{r}_x + \vec{r}_y$

$\vec{r}_x = -3\ \vec{i}$

$\vec{r}_y = 6\ \vec{j}$

$\vec{r} = -3\ \vec{i} + 6\ \vec{j}$ (m)



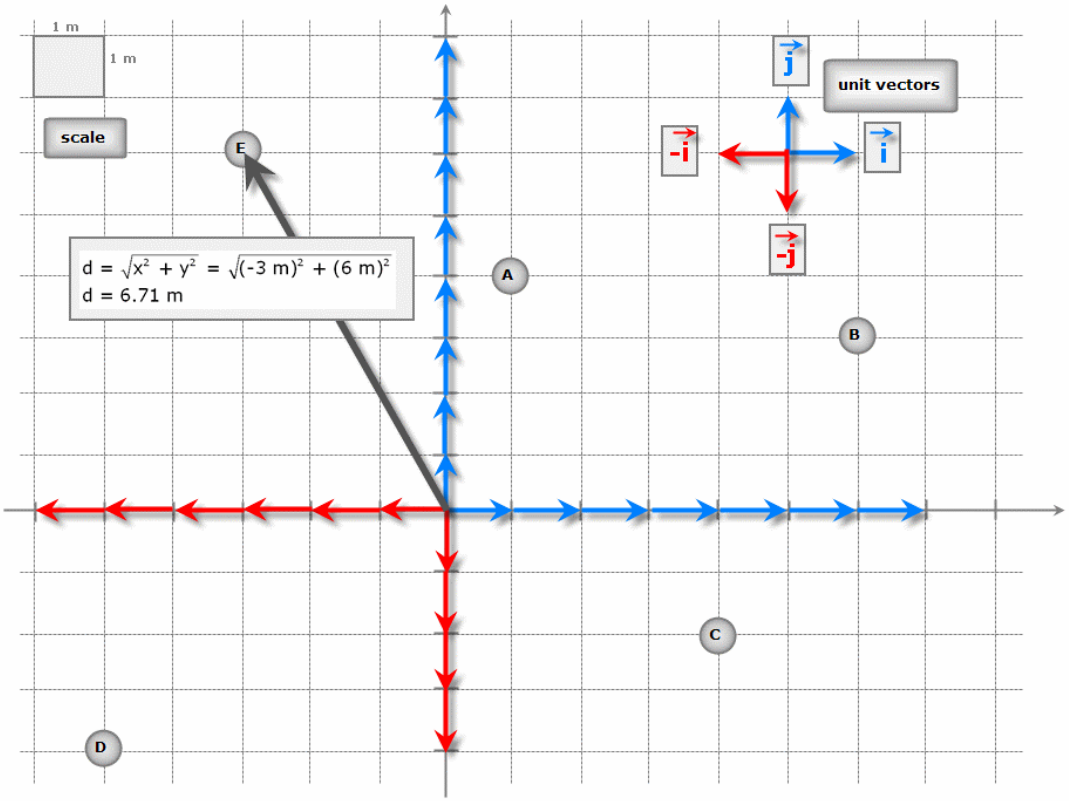
Distance

The distance from the origin to the object is the modulus of the position vector.

The distance (modulus) can be calculated from the components of the position vector. In our case the distance from the origin to the body “E” is:

$$d = \sqrt{x^2 + y^2} = \sqrt{(-3\ m)^2 + (6\ m)^2}$$

$$d = 6.71\ m$$



Exercise

Calculate the distances from the origin to the bodies “A”, “B”, “C” and “D”.

Displacement

the **displacement vector ($\Delta\vec{r}$)** indicates the sense of movement by a vector directing from the previous position to the current position.

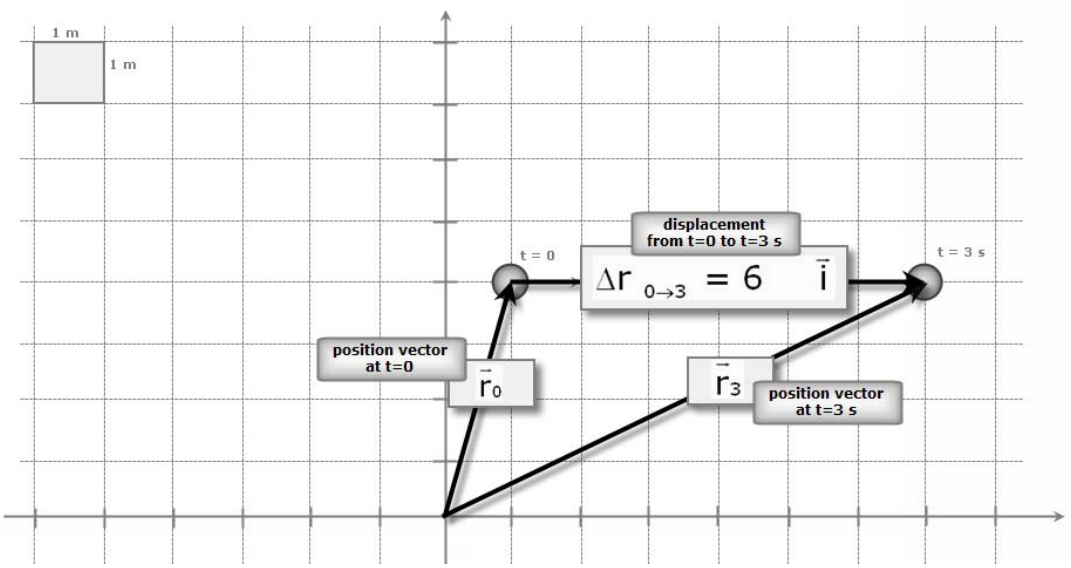
The value of the displacement is equal to the difference between final and initial position vectors

In the graphic below, the displacement of an object from t=0 to t=3 s is:

$$\vec{r}_0 = \vec{i} + 3\,\vec{j}$$

$$\vec{r}_3 = 7\,\vec{i} + 3\,\vec{j}$$

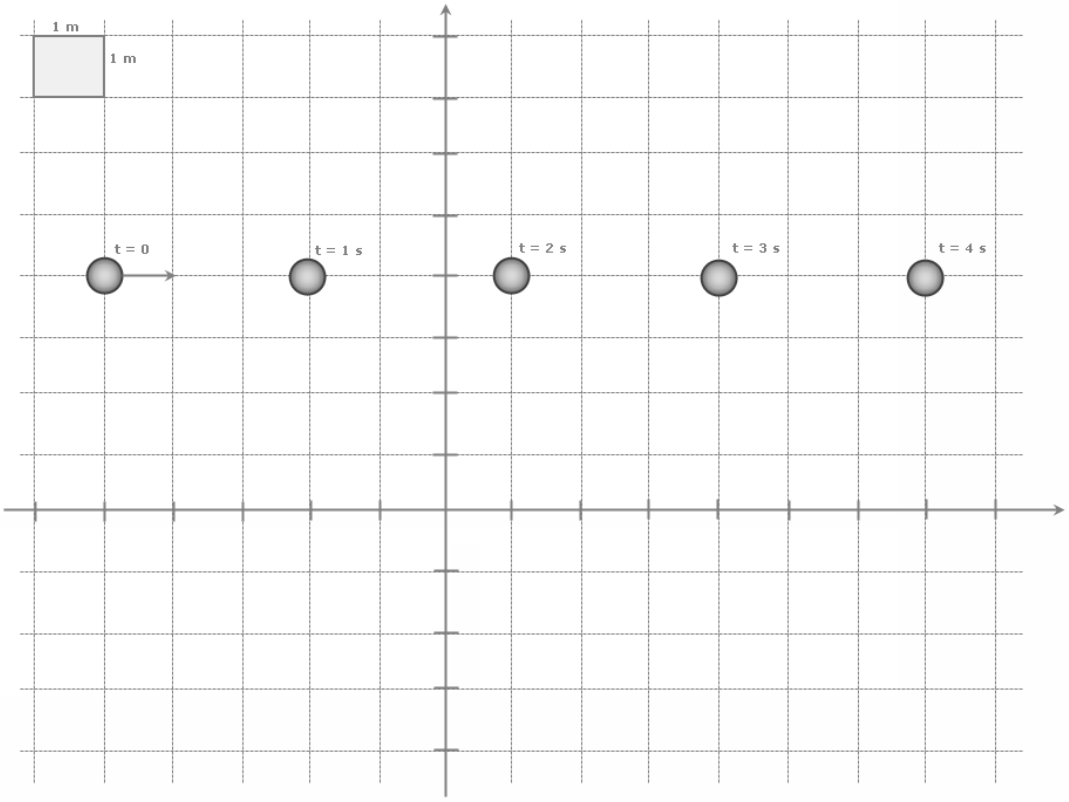
$$\Delta r_{0\to3} = \vec{r}_3 - \vec{r}_0 = (7\,\vec{i} + 3\,\vec{j}) - (\vec{i} + 3\,\vec{j}) = 6\,\vec{i}$$



Exercise

Look at the graphic below, draw and write the following expressions:

- the position vector at t=1 s
- the position vector at t=3 s
- the displacement from t=1 s to t=3 s



The equation of the position vector

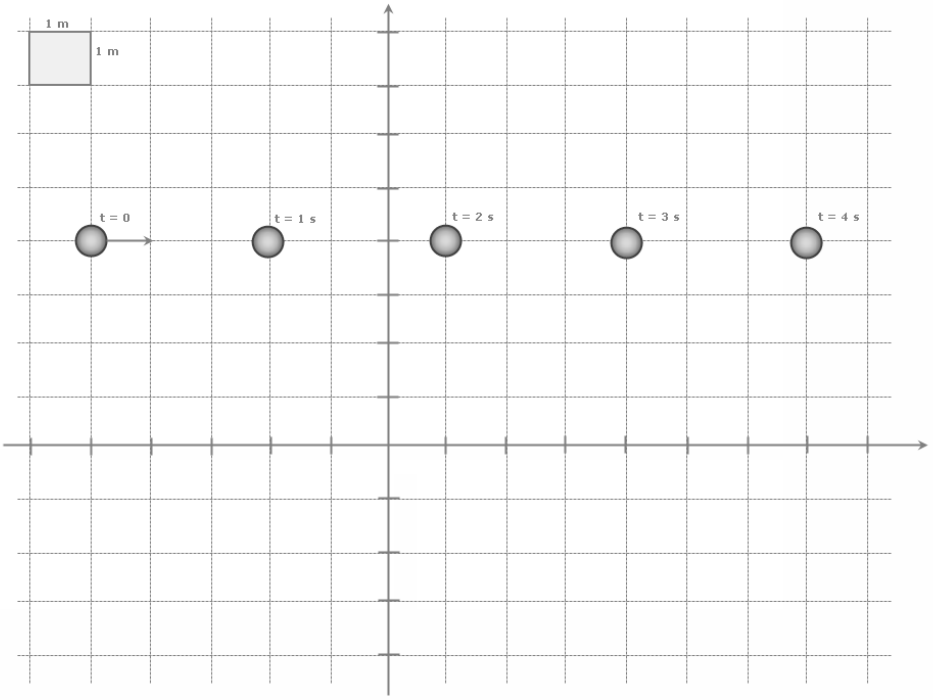
In the previous example, we specify a equation, the **equation of the position vector**, a general expression that can be used to calculate the position at any moment.

In the graphic below, if we want to determine the equation of the position vector, we need to take into account two things:

- "X" axis:
  - the position changes by +3 (to the right) every second; therefore at any interval of time the change in position is $\Delta x = 3 * t$
  - the initial position (at t=0) is $x_0 = -5$
- "Y" axis: the position is always the same, $4\ \vec{j}$

Therefore, the equation that specifies the position at any moment is:

$$\vec{r} = (-5 + 3 * t)\ \vec{i} + 4\ \vec{j}\ \ (m)$$



We can use the equation of the position vector to calculate the position at any moment. For instance, to know the position at t=2.28 s, all we need to do is to substitute the value of the time into the equation:

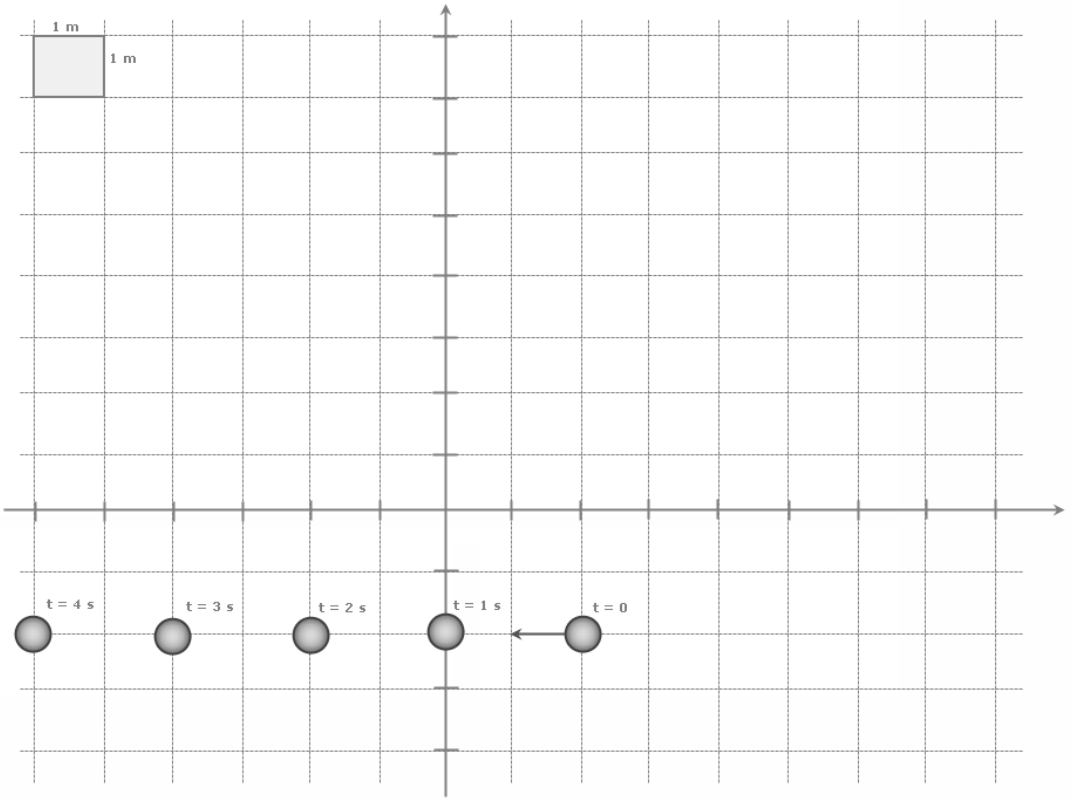
$$\vec{r} = (-5 + 3 * t)\ \vec{i} + 4\ \vec{j}\ \ (m) \xrightarrow{t=2.28}$$

$$\vec{r} = (-5 + 3 * 2.28)\ \vec{i} + 4\ \vec{j}\ \ (m) = 1.84\ \vec{i} + 4\ \vec{j}\ (m)$$

Exercise

Look at the graphic below, draw and write the following expressions:

- the position vector at t=1 s
- the position vector at t=3 s
- the displacement from t=1 s to t=3 s
- the equation of the position vector



Exercise

Look at the graphic below, draw and write the following expressions:

- the position vector at t=1 s
- the position vector at t=3 s
- the displacement from t=1 s to t=3 s
- the equation of the position vector