

<b>Topic:</b>	<b>Velocity</b>
<b>Objective:</b>	<b>FK_10_02</b>
<p><b>Given</b> an v-t graphic and some positions of a moving body  <b>the student must be capable of doing the following:</b></p> <ul style="list-style-type: none"> <li>• determine position vectors</li> <li>• determine displacements</li> <li>• determine velocities: instantaneous and average</li> </ul>	

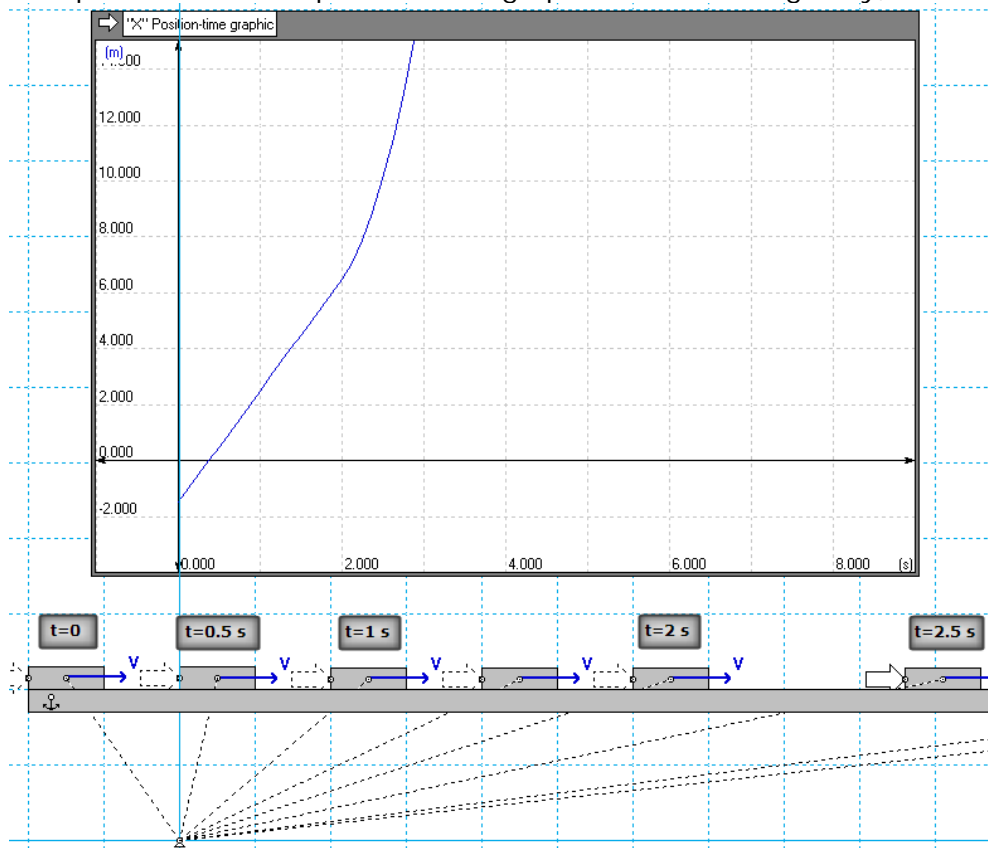
**Displacement: definition and equation**

**Displacement** ( $\Delta \vec{r}$ ) is the **change in position**. When an object changes its position from  $\vec{r}_0$  to  $\vec{r}$  the displacement can be calculated as:

$$\Delta \vec{r} = \vec{r} - \vec{r}_0$$

**Displacement: exercise**

Given the positions and the position-time graphic of this moving body,



fill in the blanks below:

**initial position vector (t=0):**  $\vec{r}_0 = -1.5 \vec{i} + 2.15 \vec{j}$  (m)

**position vector at t=2 s:**

**displacement from t=0 to t=2 s:**

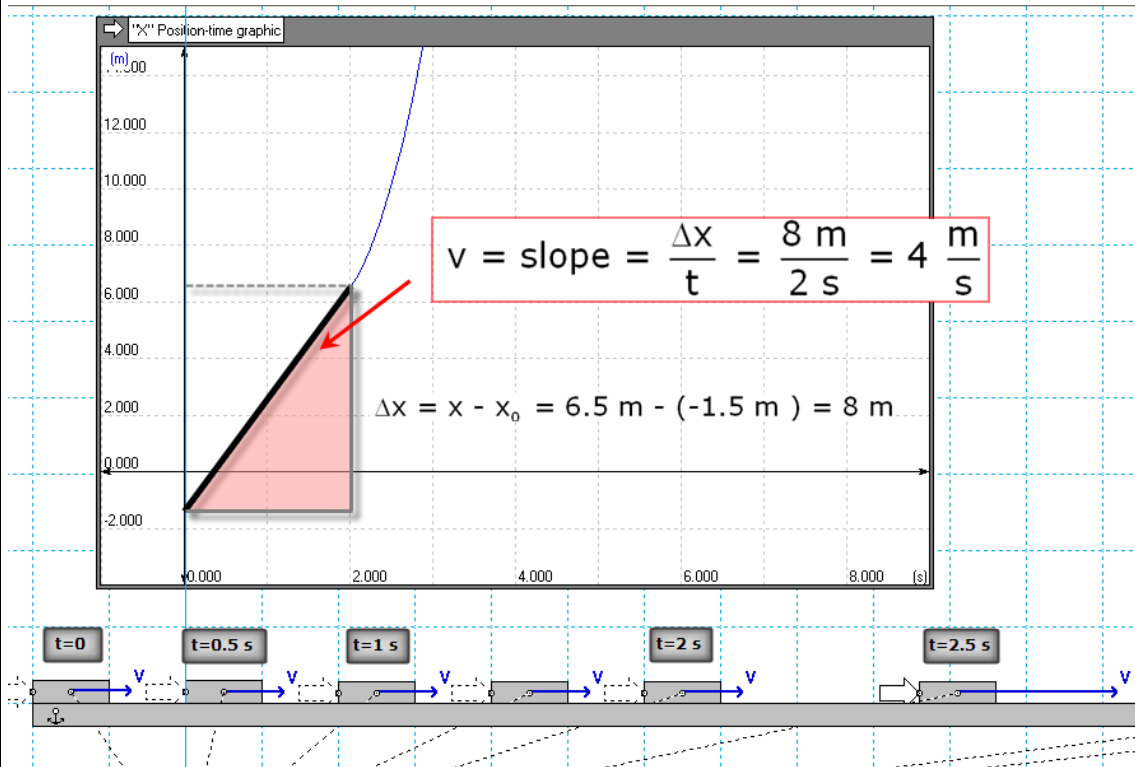
**equation of the position vector  $r=f(t)$ :**

## Constant velocity – Steady Motion

A motion in which the velocity remains constant is called steady.

**Velocity is rate of change of position.** The equation is:

$$\vec{v} = \frac{\Delta \vec{r}}{t} = \frac{\vec{r} - \vec{r}_0}{t}$$



In our case the velocity is constant during the interval from  $t=0$  to  $t=2$  s:

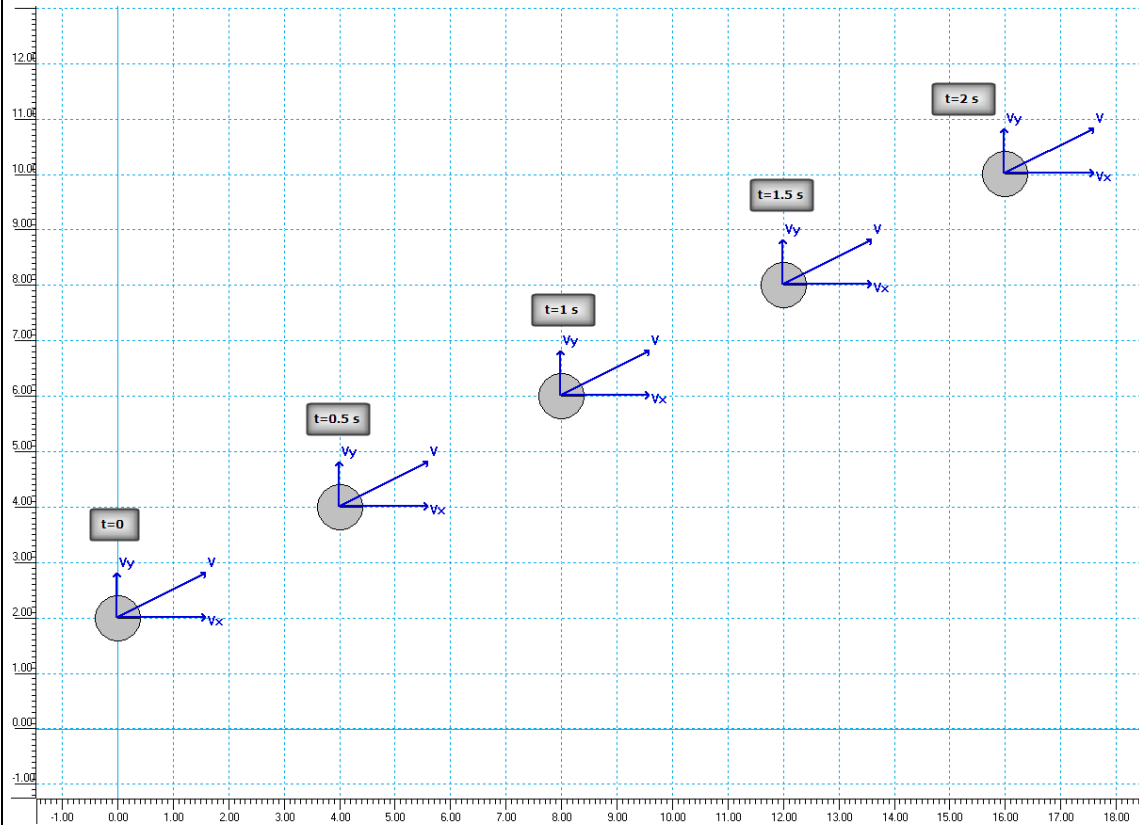
$$\vec{v} = \frac{\Delta \vec{r}}{t} = \frac{\vec{r} - \vec{r}_0}{t} = \frac{(6.5 \vec{i} + 2.15 \vec{j}) - (-1.5 \vec{i} + 2.15 \vec{j})}{2 \text{ s}} = \frac{8 \vec{i} \text{ (m)}}{2 \text{ s}} = 4 \vec{i} \text{ (m/s)}$$

The velocity has a value and a direction. The value can be calculated graphically: it is the value of the slope in a position-time graphic.

### Velocity: exercise

This body moves in two directions. In that case, we say that the velocity has two components:

$$\vec{V} = v_x \vec{i} + v_y \vec{j}$$



In this example:

- determine the components of the velocity
- write the expression of the velocity
- calculate the modulus (length) of the velocity
- calculate the angle between the velocity vector and OX

## Instantaneous velocity

The velocity measured at any given moment is called **instantaneous velocity**.

The two most common ways to determine the instantaneous velocity is

- applying the equation of the velocity
- calculating the slope of the tangent in that point

### Determination of the instantaneous velocity by applying the equation of the velocity

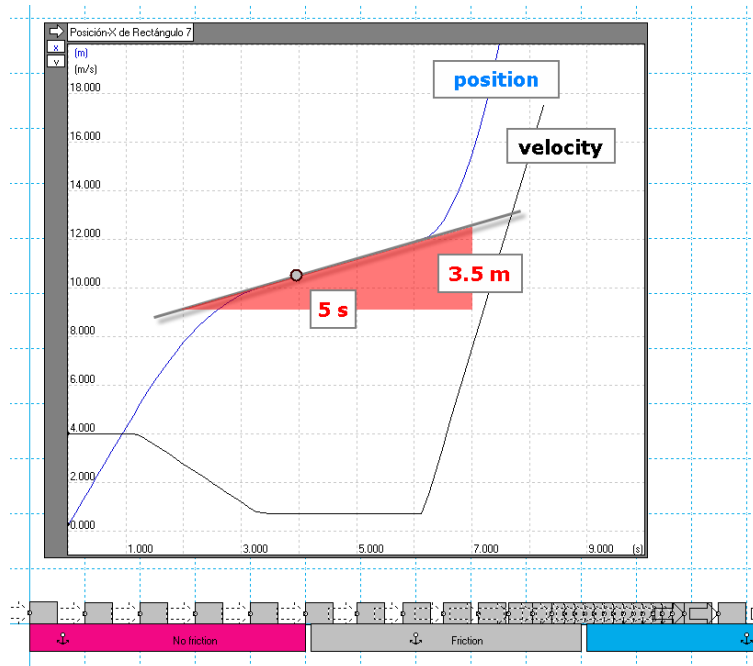
Let's suppose that the velocity of a body changes according to this equation:

$$\vec{v} = (4 + 6 \cdot t) \vec{i} \quad (\text{m/s})$$

If we want to know the velocity at  $t=4$  s, then we have to substitute that value in the equation:

$$\vec{v} = (4 + 6 \cdot t) \vec{i} \quad (\text{m/s}) \xrightarrow{t=4 \text{ s}} \vec{v} = (4 + 6 \cdot 4) \vec{i} = 28 \vec{i} \quad (\text{m/s})$$

### Determination of the instantaneous velocity graphically (calculus of the slope of the tangent in a position-time graphic)

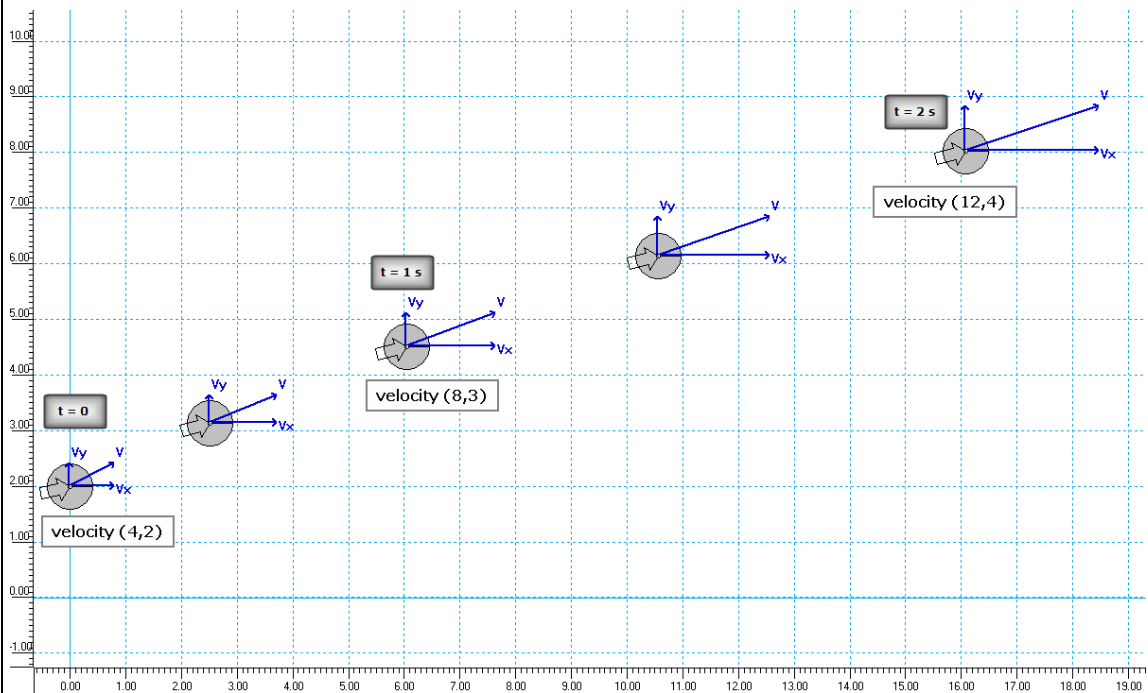


Drawing the tangent to that point we see that the slope is, approximately:

$$v = \text{slope} = \frac{3.5 \text{ m}}{5 \text{ s}} = 0.7 \frac{\text{m}}{\text{s}}$$

### Instantaneous velocity: exercise

A body is moving according to this graphic:

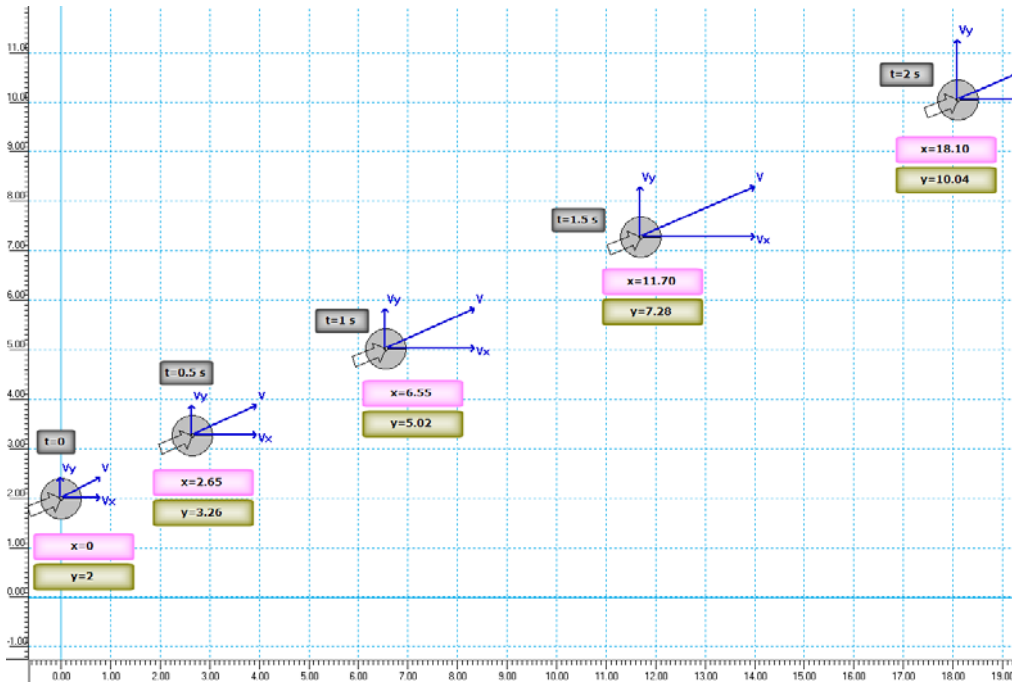


Calculate:

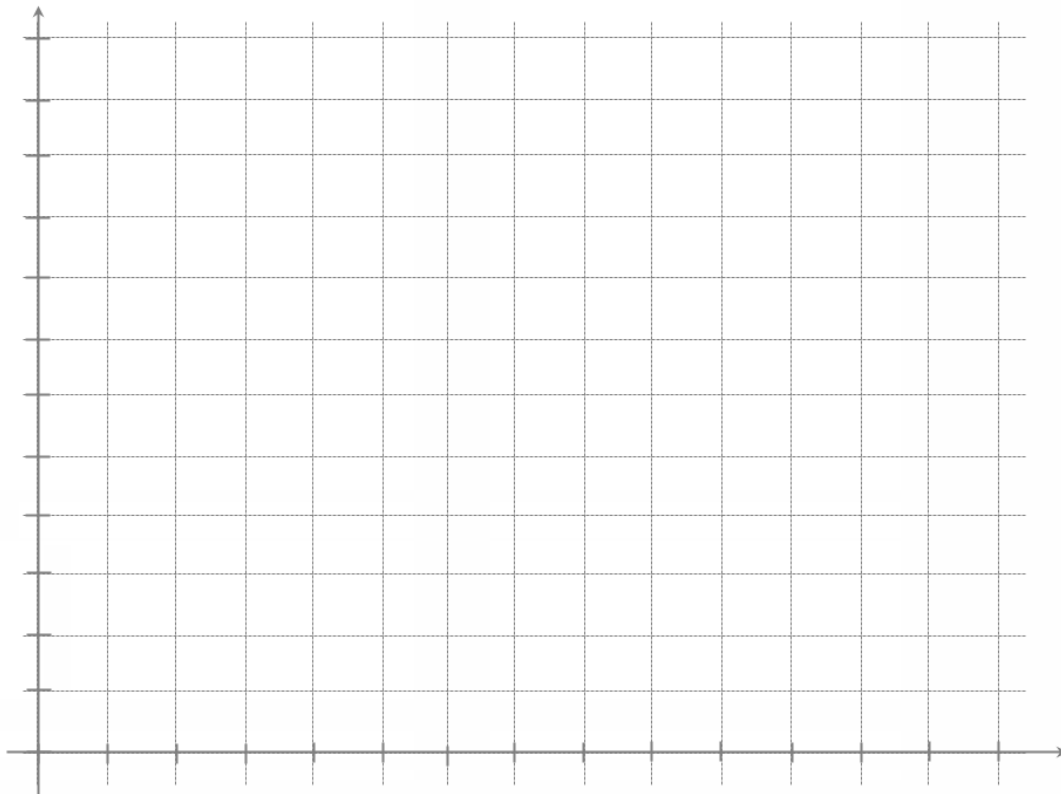
- the equation of velocity
- the expression of velocity at  $t=1.5\text{ s}$
- the modulus (length) of velocity at  $t=1.5\text{ s}$

### Instantaneous velocity: exercise

A body is moving according to this graphic:



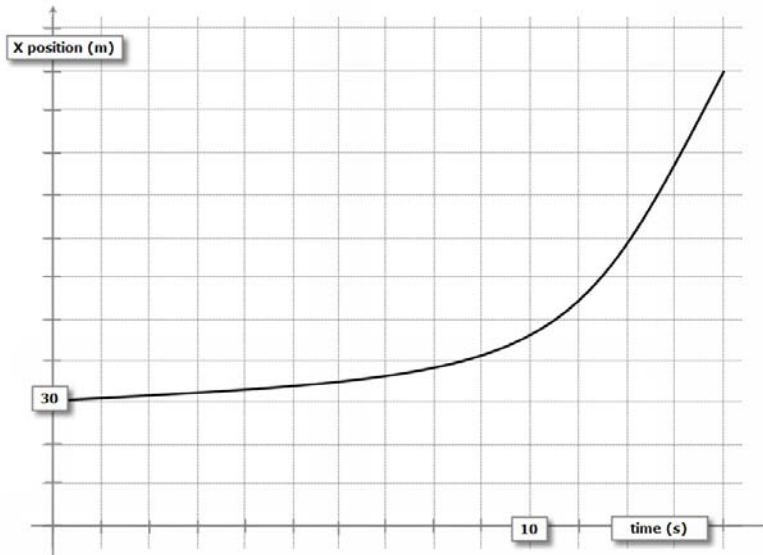
- Draw a X position – time graphic and calculate the “X” component of the velocity at  $t=1.8$  s



### Instantaneous velocity: exercise

Here you have the position (X, Y) – time graphics in order to calculate an instantaneous velocity graphically.

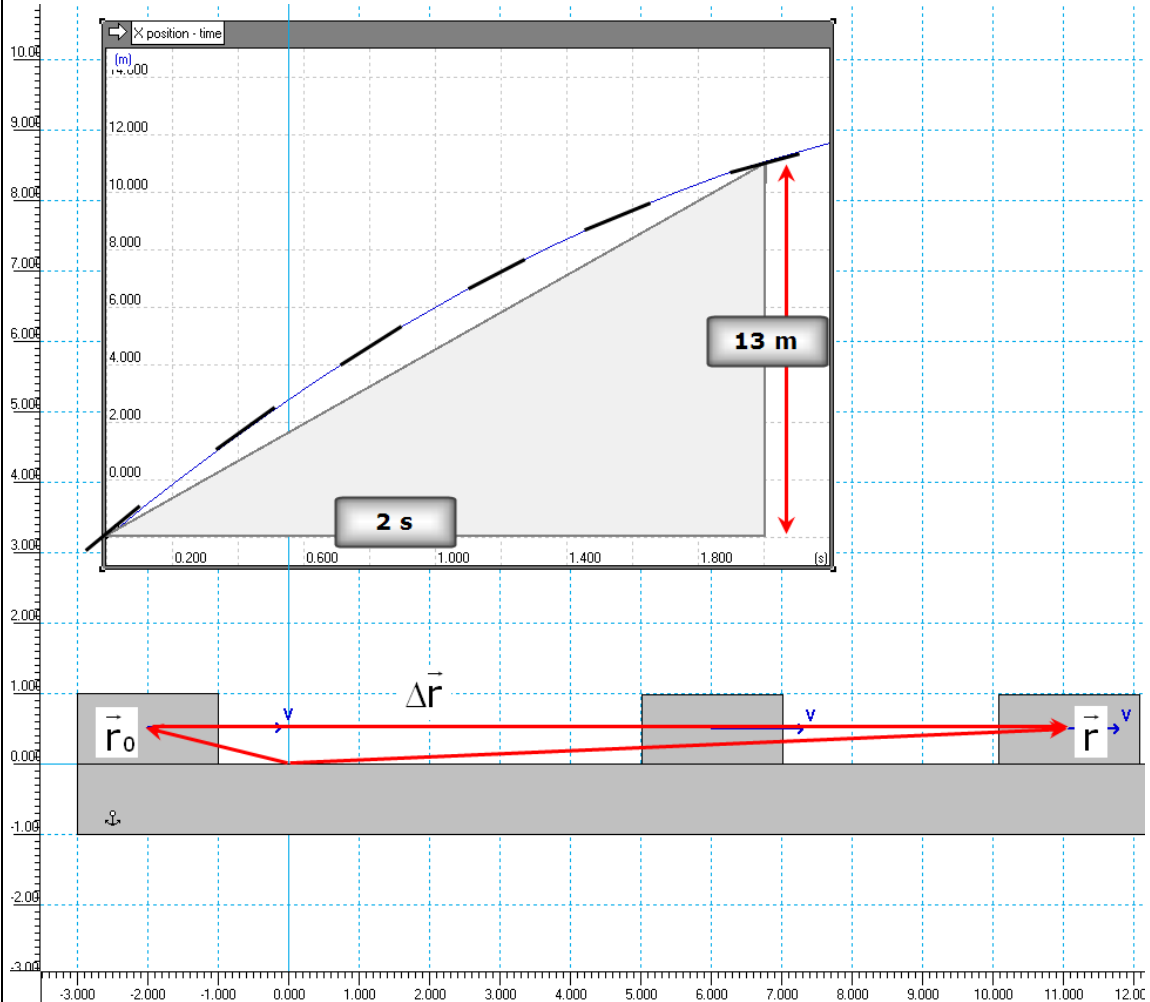
- Determine (approximately) the expression of the instantaneous velocity at  $t=10$  s.
- Draw a picture showing the position of the body at the following moments:  $t=0$ ;  $t=10$  s;  $t=14$  s



## The average velocity

The average velocity is the total displacement divided by the time interval:

$$\bar{\mathbf{v}} = \frac{\text{total displacement}}{\text{time interval}} = \frac{\Delta \vec{r}}{t} = \frac{\text{final position} - \text{initial position}}{\text{time interval}} = \frac{\vec{r} - \vec{r}_0}{t}$$



In our case, the average velocity is

$$\Delta \vec{r} = \vec{r} - \vec{r}_0 = (11 \vec{i} + 0.5 \vec{j}) - (-2 \vec{i} + 0.5 \vec{j}) = 13 \vec{i} \text{ (m)}$$

$$\bar{\mathbf{v}} = \frac{13 \vec{i} \text{ (m)}}{2 \text{ s}} = 6.5 \vec{i} \left( \frac{\text{m}}{\text{s}} \right)$$