

JAUZI LIBREA: ARIKETAK

- ① Higikazi bat 80 m -ko altueratik uzten da erortzen. Kalkulatu
 - a) zenbat denbora behar duen behera izisteko
 - b) zer abiaduraz izitsiko den luzera

- ② Higikazi bat $+20\text{ m}$ -ko altueratik gorantz botatzen da 45 m/s -ko abiaduraz. Kalkulatu
 - a) hartuko duen altuera maximoa
 - b) zenbat denbora pasako duen aizean
 - c) zer abiaduraz izitsiko den luzera

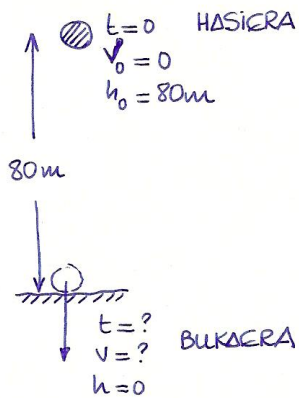
- ③ Harri bat luzetik gorantz botatzen da $+50\text{ m/s}$ -ko hasierako abiadurarekin. Kalkulatu zer abiadura izango duen $h = +80\text{ m}$ altueratik pasatzen denean.

- ④ Harri bat luzetik gorantz botatzen da eta $h = +40\text{ m}$ altueratik pasatzen denean 15 m/s -ko abiadura du. Kalkulatu:
 - a) hasierako abiadura
 - b) hartuko duen altuera maximoa

- ⑤ Harri bat luzetik gorantz botatzen da eta 60 m -ko altuera maximoa hartzen du. Kalkulatu hasierako abiadura.

JAUZI LIBREA : ARIKETEN EBAZPENAK

①



a) behera iristen denean... $h=0$ (edo $y=0$)

Eginiko desplazamendua:

$$\Delta y = y - y_0 = 0 - 80 \text{ m} = -80 \text{ m}$$

Desplazamendu hori egiteko behar duen denbora:

$$\Delta y = v_0 \cdot t - 5 \frac{\text{m}}{\text{s}^2} \cdot t^2$$

$$\Delta y = -80 \text{ m}$$

$$v_0 = 0$$

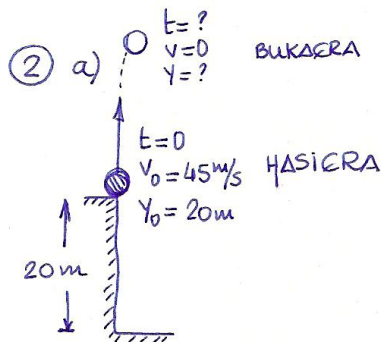
$$-80 \text{ m} = -5 \frac{\text{m}}{\text{s}^2} \cdot t^2 \rightarrow t^2 = \frac{-80 \text{ m}}{-5 \text{ m/s}^2} = 16 \text{ s}^2$$

$$t = \sqrt{16 \text{ s}^2} = 4 \text{ s} \text{ behar duen denbora lurrera iristeko}$$

g) Abiadura lurrera iristen denean

$$v = v_0 - 10 \frac{\text{m}}{\text{s}^2} \cdot t \rightarrow v = -40 \frac{\text{m}}{\text{s}}$$

beherantz



Altuera maximoa: y edo h

$$\Delta y = v_0 \cdot t - 5 \frac{\text{m}}{\text{s}^2} \cdot t^2$$

$$\downarrow v_0 = 45 \text{ m/s}$$

$$\Delta y = 45 \frac{\text{m}}{\text{s}} \cdot t - 5 \frac{\text{m}}{\text{s}^2} \cdot t^2 \rightarrow 2 \text{ ezezagun}$$

Abiaduraren ekuaziotik denbora kalkulatu daiteke:

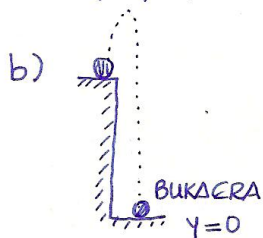
$$v = v_0 - 10 \frac{\text{m}}{\text{s}^2} \cdot t \rightarrow 0 = 45 \frac{\text{m}}{\text{s}} - 10 \frac{\text{m}}{\text{s}^2} \cdot t \rightarrow t = \frac{45 \text{ m/s}}{10 \text{ m/s}^2} = 4.5 \text{ s}$$

"t" jakin ondoren, desplazamendua kalkulatu daiteke:

$$\Delta y = 45 \frac{\text{m}}{\text{s}} \cdot 4.5 \text{ s} - 5 \frac{\text{m}}{\text{s}^2} \cdot (4.5 \text{ s})^2 = 202.5 \text{ m} - 101.25 \text{ m} = 101.25 \text{ m}$$

Altuera maximoa (y):

$$\Delta y = y - y_0 \rightarrow y = \Delta y + y_0 = 101.25 \text{ m} + 20 \text{ m} = \boxed{121.25 \text{ m}}$$



denbora airean (t)

$$\Delta y = y - y_0 = 0 - 20 \text{ m} = -20 \text{ m}$$

Desplazamenduaren ekuazioa erabiliz:

$$\Delta y = v_0 \cdot t - 5 \frac{\text{m}}{\text{s}^2} \cdot t^2$$

$$\downarrow \Delta y = -20 \text{ m}$$

$$\downarrow v_0 = 45 \text{ m/s}$$

$$-20 \text{ m} = 45 \frac{\text{m}}{\text{s}} \cdot t - 5 \frac{\text{m}}{\text{s}^2} \cdot t^2 \rightarrow -20 = 45t - 5t^2 \rightarrow$$

bigarren mailako ekuazioa

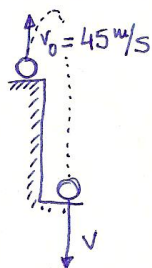
$$\rightarrow 5t^2 - 45t - 20 = 0 \rightarrow t^2 - 9t - 4 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{9 \pm \sqrt{81 + 16}}{2}$$

$$\nearrow t < 0$$

$$\searrow t = \frac{9 + 9.85}{2} = \boxed{9.43 \text{ s}} \text{ airean}$$

② c) Abiadura lurrean

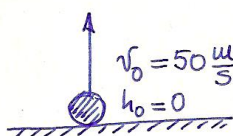


$$v = v_0 - 10 \frac{\text{m}}{\text{s}^2} \cdot t$$

9'43 s airean pasa duenez, $t = 9'43 \text{ s}$

$$v = 45 \frac{\text{m}}{\text{s}} - 10 \frac{\text{m}}{\text{s}^2} \cdot 9'43 \text{ s} = -49'3 \frac{\text{m}}{\text{s}}$$

beherantz



Desplazamenduaren ekuazioa erabiliz:

$$\Delta y = v_0 \cdot t - 5 \frac{\text{m}}{\text{s}^2} \cdot t^2$$

$\Delta y = y - y_0 = 80 \text{ m} - 0 = 80 \text{ m}$

Bigarren mailako ekuazioa askatuko dugu:

$$80 = 50t - 5t^2 \rightarrow 5t^2 - 50t + 80 = 0 \quad \text{sinplifikatuz}$$

$$\rightarrow t^2 - 10t + 16 = 0 \rightarrow t = \frac{10 \pm \sqrt{100 - 64}}{2} = \frac{10 \pm 6}{2} \begin{matrix} \nearrow 8 \text{ s} \\ \searrow 2 \text{ s} \end{matrix}$$

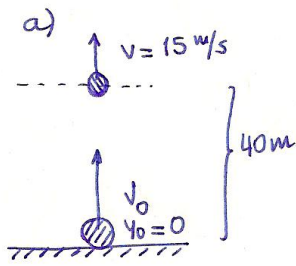
Abiaduraren kalkulua: $v = v_0 - 10 \frac{\text{m}}{\text{s}^2} \cdot t$

$$t = 8 \text{ s} \rightarrow v =$$

$$v = 50 \frac{\text{m}}{\text{s}} - 10 \frac{\text{m}}{\text{s}^2} \cdot 8 \text{ s} = -30 \frac{\text{m}}{\text{s}} \rightarrow \text{beherantz doa } 8. \text{ segunduan}$$

$$t = 2 \text{ s} \rightarrow v = 50 \frac{\text{m}}{\text{s}} - 10 \frac{\text{m}}{\text{s}^2} \cdot 2 \text{ s} = +30 \frac{\text{m}}{\text{s}} \rightarrow \text{gorantz doa } 2. \text{ segunduan}$$

4)



Desplazamenduaren ekuazioa erabiliz:

$$\Delta y = v_0 \cdot t - 5 \frac{m}{s^2} \cdot t^2 \rightarrow \text{ekuazioa 2 ezezagunekin}$$

$$\Delta y = y - y_0 = 40m$$

Abiaduraren ekuazioa erabiliz:

$$v = v_0 - 10 \frac{m}{s^2} \cdot t \rightarrow 15 \frac{m}{s} = v_0 - 10 \frac{m}{s^2} \cdot t \rightarrow \text{ekuazioa 2 ezezagunekin}$$

Sistema bat daukagu:

$$\begin{cases} 40 = v_0 \cdot t - 5t^2 & \xrightarrow{\text{ORDEZKATUZ}} 40 = (15 + 10t) \cdot t - 5t^2 \\ 15 = v_0 - 10t \rightarrow v_0 = 15 + 10t & \end{cases}$$

$$40 = 15t + 10t^2 - 5t^2 \rightarrow$$

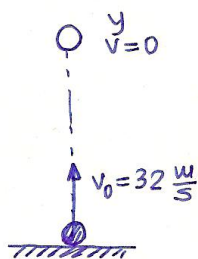
$$\rightarrow 5t^2 + 15t - 40 = 0 \xrightarrow{\text{SINPLIFIKATUZ}} t^2 + 3t - 8 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{9 + 32}}{2} = \frac{-3 \pm \sqrt{41}}{2} \begin{cases} t < 0 \\ t = 1.70s \end{cases}$$

Abiaduraren ekuazioan denbora ordezkatuz hasierako abiadura kalkulatuko dugu:

$$15 \frac{m}{s} = v_0 - 10 \frac{m}{s^2} \cdot 1.70s = v_0 - 17 \frac{m}{s} \rightarrow v_0 = 32 \frac{m}{s}$$

b) Altuera maximoa



$$v = v_0 - 10 \frac{m}{s^2} \cdot t \rightarrow 0 = 32 \frac{m}{s} - 10 \frac{m}{s^2} \cdot t \rightarrow$$

$$\rightarrow 10 \frac{m}{s^2} \cdot t = 32 \frac{m}{s} \rightarrow t = \frac{32 \frac{m}{s}}{10 \frac{m}{s^2}} = 3.2s$$

behar du gora izisteko

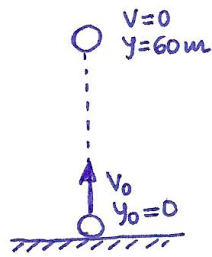
$$\Delta y = v_0 \cdot t - 5 \frac{m}{s^2} \cdot t^2 \rightarrow \Delta y = 32 \frac{m}{s} \cdot 3.2s - 5 \frac{m}{s^2} \cdot (3.2s)^2 =$$

$$= 102.4m - 51.2m = 51.2m$$

$$\Delta y = y - y_0 \rightarrow 51.2m = y - 0 \Rightarrow y = 51.2m$$

altuera maximoa

5



Desplazamenduaren ekuazioa erabiliz:

$$\Delta y = v_0 \cdot t - 5 \frac{\text{m}}{\text{s}^2} \cdot t^2 \rightarrow 60 \text{m} = v_0 \cdot t - 5 \frac{\text{m}}{\text{s}^2} \cdot t^2$$

2 ezezagun

$$\Delta y = y - y_0 = 60 \text{m}$$

Abiaduraren ekuazioa erabiliz:

$$v = v_0 - 10 \frac{\text{m}}{\text{s}^2} \cdot t \rightarrow 0 = v_0 - 10 \frac{\text{m}}{\text{s}^2} \cdot t \quad 2 \text{ ezezagun}$$

Honako sistema hau askatuko dugu:

$$\begin{cases} 0 = v_0 - 10 \cdot t \rightarrow v_0 = 10 \cdot t \\ 60 = v_0 \cdot t - 5 t^2 \end{cases} \xrightarrow{\text{ORDEZKATUZ}} 60 = (10 \cdot t) \cdot t - 5 t^2 = 10 t^2 - 5 t^2 = 5 t^2$$

$$t^2 = \frac{60}{5} = 12 \rightarrow t = \sqrt{12} = 3\sqrt{3} \text{ s} \quad \text{behaz du lurretik gora izisteko}$$

Denbora abiaduraren ekuazioan ordezkatuz hasierako abiadura kalkulatuko dugu:

$$v_0 = 10 \frac{\text{m}}{\text{s}^2} \cdot t = 34\sqrt{3} \frac{\text{m}}{\text{s}}$$